GG CO MILLO Teacher Sourcebook

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ORIGO

Sample Unit

Working with Multiplication and Division (Factors and Multiples)



Content Outcomes

NS 2.3 Uses mental and informal written strategies for multiplication and division

Mathematical Background

In this unit, students investigate number patterns involving multiples. On a blank hundred chart, they locate numbers that they say when they count by 2, 3, 4 or 5, and then look for patterns in a table of multiplication facts. The build-up and build-down strategies are revised early in the unit and then reinforced later with examples beyond the range of number facts. Students work with multiplication examples involving three factors and learn that multiplying the numbers in any order does not change the answer. Confidence with the build-up and build-down strategies and awareness of multiplication in any order will help students when they multiply large numbers. Students also explore methods to find factors of two-digit numbers. These activities help reinforce the link between multiplication and division.

Lesson Overview

- 37.1 Investigating Patterns Involving Multiples
- 37.2 Using a Strategy that Builds Up or Down from a Known Multiplication Fact
- 37.3 Multiplying with Three Factors
- 37.4 Investigating Methods to Find Factors
- 37.5 Identifying Two-Digit and Three-Digit Multiples of 2, 3, 4 or 5

Investigation: Which nine different numbers would you want to have on your multiplication facts Bingo card?

Language

Students will use and develop the following language: halve, double, build up, build down, multiple, factor

Sequence of	f related	units t	for Sta	ige 2
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	Stage 2A	Stage 2B
NS 2.3	5 8 12 14 19 26 27 30	37 40 41 44 46 51 52 58 59 62



Content Indicators On completion of this unit, the students should be able to A identify and describe numerical patterns in a multiplication chart B use known tens and fives multiplication facts to help work out the facts involving nines and sixes respectively C calculate the product of three single-digit factors, e.g. 4 × 3 × 7 D list pairs of factors that share the same product E identify two- and three-digit multiples of 3 and 4

Techniques

The following tools can be used to assess the content indicators.

1. Investigation A D

The Investigation is located after Lesson 5.

2. Written Test B C D E

Allow time for the students to complete the written test for Unit 37. See page 5 of the *GO Check* assessment book. Consider administering the test one or two weeks after completion of the unit.

3. Student Journal Page

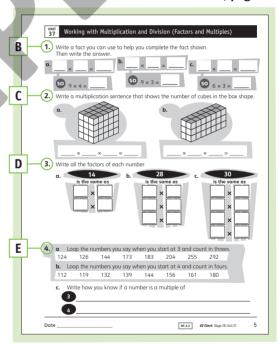
Refer to GO Maths student journal page 22.

4. Diagnostic Probe D

Ask the student to list the pairs of factors that make 12.

If the student is successful, repeat the activity for the number 40.

GO Check, page 5



Working Mathematically

The processes related to the working mathematically outcomes are embedded in all *GO Maths* activities. The following pointers should help you assess the students' demonstration of these processes.

Questioning (WMS 2.1)	Listen to the questions students pose as they try to predict numbers in patterns that involve multiplication.
Applying Strategies (WMS 2.2)	Observe the way students use known multiplication facts to work out unknown facts. Do they build up (down)? Do they halve (double)?
Communicating (WMS 2.3)	Check the language students use when talking about multiplication: are they using terms like 'factor' and 'multiple' correctly?
Reasoning (WMS 2.4)	Listen as students explain how strategies like building up or down and halving or doubling make computation easier.
Reflecting (WMS 2.5)	Find out if students see the link between multiplying by three factors and finding the volume of a rectangular prism.

Recording

Content

Record achievement of the content indicators in the box(es) for this unit alongside the relevant outcome(s) in the Progress Record. This can be found on page v of each student's *GO Check* assessment book. Alternatively, enter results into the *GO Chart Electronic Progress Record*.

	Stage 2A	Stage 2B
NS 2.3	5 8 12 14 19 26 27 30	37 40 41 44 46 51 52 58 59 62

Working Mathematically

Record significant observations in the Progress Record on page vi of each student's *GO Check* assessment book. Alternatively, enter more detailed notes into the *GO Chart Electronic Progress Record*.

Intervention

For students who need extra assistance, revisit the relevant parts of this unit or check for suitable activities from the Related Classroom Resources section in the back of this sourcebook.



Investigating Patterns Involving Multiples

Materials

- GO Maths student journal, page 22
- Blank hundred chart copied onto an overhead transparency (use Blackline Master 4)
- · Overhead projector
- Counters R

Optional

- 1 blank hundred chart (use Blackline Master 4) for each group of students
- 3 cubes **R** for each group of students with the following on the faces:
 - Cube 1: 1, 2, 3, 4, 5, 6
 - Cube 2: 3, 4, 5, 6, 7, 8
 - Cube 3: 4, 5, 6, 7, 8, 9

Daily Computation Practice

Say any multiplication fact from the set of twos, fours or fives facts, including the turnarounds. Have the students write the answers.

Consolidation

Give each group of at least four students a blank hundred chart, counters and the cubes. Have them form teams and take turns to roll all three cubes, choose two numbers to multiply together and place a counter on the answer. The first team with three counters in a row wins the game.

In this lesson, students use a blank hundred chart to identify multiples of 10, 5, 4 and 3. They then search for patterns in a chart of all multiplication facts.

Daily Number Sense

Write the number fact $4 \times 6 =$ on the board. Say: Imagine you forgot the answer to this fact. How could you work out the answer? Invite individuals to describe their thinking (e.g. 'Double 6 is 12 and double 12 is 24'). Ask: What other facts can you work out using $4 \times 6 = 24$? Encourage examples such as $6 \times 4 =$ (the turnaround), $8 \times 6 = 48$ (double), $5 \times 6 = 30$ (build up 6), $3 \times 6 = 18$ (build down 6), $7 \times 4 = 28$ (build up 4) and $5 \times 4 = 20$ (build down 4).

Activity

- 1. Display the blank hundred chart on the overhead projector and ask: *How many squares are in this grid? How do you know?* Bring out the fact that there are ten rows of ten. Then ask: *If we count in steps of ten and place counters where we land, what numbers will the counters cover?* Have volunteers place counters on spaces and explain how they identified the numbers. For example, a student could put a counter in the sixth space down the right-hand column and say the number is 60 because it is six jumps of ten. They can also use information from spaces already identified with counters. Encourage students to choose numbers randomly rather than starting with 1 ten, 2 tens, 3 tens etc.
- 2. Remove the counters and repeat the activity for numbers the students would say if they count by fives. They can identify any multiples of 5 on the grid.
- 3. Remove the counters and repeat the activity for numbers they say when they count by fours and then by threes. Encourage students to use a multiple already identified (e.g. 4 fives) to help identify other spaces that are multiples of 4.
- 4. Discuss the grid at the top of page 22 of the *GO Maths* student journal. Ask: *What numbers do you see in the fifth row?* Establish that these are multiplication facts (or multiples of 5). Then have the students work independently to complete the page.

Reflection

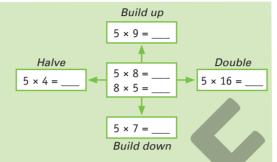
- 1. Discuss the students' answers to page 22 of the GO Maths student journal.
- 2. Show the overhead transparency again and place a counter on any space. Ask: *Is this a number you say when you count by fives? How do you know?* Invite students to describe their thinking. Repeat the discussion for the counter on the same space and counting by threes and then fours.

Using a Strategy that Builds Up or Down from a Known Multiplication Fact

This lesson reinforces the very useful strategy of using a known fact to help work out the answer to a nearby fact.

Daily Number Sense

Draw this diagram in a place where it can be left for the rest of this unit. Ask the students to work out the answer for the facts in the centre space and then explain how the other number sentences are related to the centre facts. Erase all of the number sentences and have a



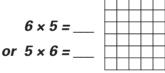
volunteer write a new fact and its turnaround in the centre. Ask the students to suggest number sentences for each of the other spaces that they can work out using either of the two facts in the centre. The sentences could involve facts or extensions of the facts.

Activity

- 1. Cut out a 3-by-6, a 6-by-6 and a 7-by-10 array from the blank hundred charts. Fold along one row of each array in the following manner:
 - one row of 6 in the 3-by-6 array
 - one row of 6 in the 6-by-6 array
 - one row of 7 in the 7-by-10 array

Each array can then be used to show related facts, as shown below for the 6-by-6 array.





- 2. Write 6 x 6 = ___ and 6 x 5 = ___ on the board. Ask: Which of these facts can you answer quickly? How can you use 6 x 6 to help work out the answer to 6 x 5? Show the students the 6-by-6 array picture and demonstrate how the fact they can answer quickly can be used to work out the other fact. At this stage, the students may be able to answer both facts quickly, but it is important to reinforce the strategy.
- 3. Repeat the discussion for $3 \times 6 =$ ___ and $2 \times 6 =$ ___, then for $7 \times 10 =$ ___ and $7 \times 9 =$ ___.
- 4. Have the students work independently to complete page 23 of the *GO Maths* student journal.

Reflection

- 1. Discuss the students' answers to page 23 of the GO Maths student journal.
- 2. Write the following facts on the board: $3 \times 7 =$ ___, $8 \times 9 =$ ___ and $6 \times 7 =$ ___. Invite individuals to identify a fact they could use to help work out each answer. Encourage them to write the fact and describe the thinking they would use.

27.2

Materials

- GO Maths student journal, page 23
- 2 blank hundred charts (use Blackline Master 4)

Optiona

- 1 blank hundred chart (use Blackline Master 4) for each group of students
- 3 cubes from the previous lesson for each group of students
- Counters R
- Fundamentals (Ages 9–10), pages 36–39

Daily Computation Practice

Say any multiplication fact from the set of fours, fives, eights or nines facts, including the turnarounds. Have the students write the answers.

Consolidation

- Students could continue to play the game introduced in the previous Consolidation activity.
- 2. Have the students play 'Multiplication Mania' from Fundamentals (Ages 9–10), pages 36–39.



Multiplying with Three Factors

Materials

- GO Maths student journal, page 24
- · Base-10 ones blocks

Optional

 3 cubes from 37.1 for each group of students

Daily Computation Practice

Say any multiplication fact from the set of threes, fives, sixes, eights or nines facts, including the turnarounds. Have the students write the answers.

Consolidation

In groups, have the students roll all three cubes and multiply the numbers rolled. They score:

- 1 point for a product more than 100
- 3 points for a product from 50–100
- 5 points for a product less than 50

It is not necessary for the students to calculate the exact answer each time to determine the points.

Extension

Challenge confident students to write all of the different number sentences with three factors that total 60. They can use ones blocks to help. In this lesson, students use base-10 ones blocks to build rectangular prisms and use multiplication to work out the total number of cubes.

Daily Number Sense

Repeat the Daily Number Sense discussion from the previous lesson with **4** × **8** = __ and **8** × **4** = ___ in the centre space. Make sure the students realise that they can work with either number sentence to build up or down, or double or halve.

Activity

- 1. Ask each student to build a rectangular prism with 36 base-10 ones blocks (cubes). Challenge them to find all of the different prisms they can build with all dimensions greater than one centimetre. (There are three solutions: 2 cm by 2 cm by 9 cm, 2 cm by 3 cm by 6 cm, and 3 cm by 3 cm by 4 cm.) Have the students record their findings in any form they choose.
- 2. Ask individuals to describe the prisms and record the dimensions on the board. Draw one example on the board and label the dimensions as shown on page 24 of the *GO Maths* student journal. Ask: *How do you know this prism has 36 cubes in it?* Bring out the fact that a face is an array and each face is reproduced two or more times to build the prism. Write one or more multiplication number sentences for the prism.
- 3. Have the students move into groups and give each group a different total of cubes to construct a prism. Use these totals: 24 cubes, 28 cubes, 32 cubes, 40 cubes and 45 cubes. As each group finds one solution, have a confident student draw and label the dimensions of the prism on the board.
- 4. Discuss each solution and write at least one multiplication sentence on the board that uses the dimensions of the prism to verify the total number of cubes.
- 5. Have the students work independently to complete page 24 of the *GO Maths* student journal.

Reflection

- 1. Discuss the students' answers to page 24 of the GO Maths student journal.
- 2. Ask: What is another order we could use to write the factors for the first prism (30 cubes)? What two numbers would you multiply first? Does the order that you multiply 2, 5 and 3 make any difference? Reinforce that numbers can be multiplied in any order.

Investigating Methods to Find Factors

In this lesson, students find the dimensions of arrays that can be made with a specified number of cubes. The dimensions of the arrays are used to find factors of the number.

Daily Number Sense

Repeat the Daily Number Sense discussion from 37.2 with $\mathbf{8} \times \mathbf{6} = \underline{}$ and $\mathbf{6} \times \mathbf{8} = \underline{}$ in the centre space.

Activity

- 1. Say: Count out 20 cubes each. What different-shaped rectangles can you make with that number? What number sentences can you write for each rectangle? Have the students work in groups to find all of the rectangles and encourage them to use a multiplication number sentence to describe each rectangle (1 × 20 = 20, 2 × 10 = 20 and 4 × 5 = 20 some groups might write the turnaround sentences). Remind them that one row of 20 cubes is a rectangle.
- 2. Invite individuals to write the number sentences on the board.
- 3. Repeat Steps 1 and 2 for 32 cubes. As the students are writing the sentences on the board, ask: *How did you work out the numbers for another sentence?* Encourage the students to explain how they tried each number in order as a divisor or used strategies such as doubling and halving.
- 4. Have the students work with a partner and some cubes to complete page 25 of the *GO Maths* student journal.

Reflection

- 1. Briefly check the students' answers to page 25 of the GO Maths student journal.
- 2. Ask: What whole numbers were used to form the multiplication number sentences to make 15? (1, 3, 5, 15.) Have the students identify the four numbers and write them on the board. Explain that these numbers are called 'factors' (see the 'Did You Know?' note on the right). Repeat the question for one number in each row of examples on page 25.



Materials

- GO Maths student journal, page 25
- GO Figure computation practice book, page 9
- Base-10 ones blocks or square tiles

Teaching Note

Grid paper could be used for this activity rather than ones blocks or tiles. Blocks involve a third dimension and students may not focus on the rectangles they are making. However, blocks and tiles have the advantage of allowing movement and experimentation.

Daily Computation Practice

Have the students complete page 9 of the *GO Figure* computation practice book. Encourage them to time how long it takes them to write the answers. They should add five seconds for each incorrect answer.

Did You Kunu?

The word 'factor' is from the Latin factus, which means 'maker'. In this sense, two or more factors 'make' a product when multiplied together. The words 'factor' and 'factory' have the same origin. A factory makes a product in the same way that factors are used to make products.



Identifying Two-Digit and Three-Digit Multiples of 2, 3, 4 or 5

Materials

- GO Maths student journal, page 26
- GO Figure computation practice book, page 10
- Blank hundred chart copied onto an overhead transparency (use Blackline Master 4)
- · Overhead projector
- Non-permanent marker

Daily Computation Practice

Have the students complete page 10 of the *GO Figure* computation practice book. Encourage them to time how long it takes them to write the answers. They should add five seconds for each incorrect answer.

Did You Know?

The root word for 'multiple' means 'many fold'. Thus, a multiple is a number repeated many times.

In this lesson, students are encouraged to use patterns and logical reasoning to develop ways to identify numbers that are multiples of 2, 3, 4 or 5. At the same time, they are informally developing tests for divisibility.

Daily Number Sense

Repeat the Daily Number Sense discussion from 37.2. Invite students to suggest the multiplication facts to write in the centre space.

Activity

- 1. Ask: What are some numbers you know you will say if you start at 2 and count in steps of two? Invite individuals to suggest numbers less than 30 and explain their thinking. Write each number on the board. If necessary, ask volunteers to check by counting by twos.
- 2. Ask students to give any numbers less than 100 they will say when they count by twos and describe the thinking they used.
- 3. Ask: What name do we use for the numbers we say when we count by twos? (Multiples see the 'Did You Know?' note below left.) What are the multiples of 5 up to 100? What multiples of 5 will you say if you count by twos? Display the overhead transparency and have volunteers take turns to write the multiples of 5 that are also multiples of 2 in the correct spaces on the chart. (The multiples of 10 should be written in the right-hand column.)
- 4. Have the students work independently to complete Question 1 on page 26 of the GO Maths student journal. Erase the numbers on the transparency. Then invite individuals to write their answers to Question 1a on the transparency and describe how they used these numbers to work out which of the numbers in Question 1b were multiples of 3.
- 5. Have the students complete Questions 2 and 3 on page 26 of the *GO Maths* student journal.

Reflection

- 1. Discuss the students' answers to Questions 2 and 3 on page 26 of the *GO Maths* student journal.
- 2. Ask: How do you know that 120 (150, 160 etc.) is a multiple of 3 (multiple of 4)? Encourage students to explain how they used the pattern for multiples of 10 less than 100 or some other observation, such as their knowledge of number facts. Then discuss questions such as: If you know 120 is a multiple of 3 (multiple of 4), what other nearby numbers are also multiples of 3 (multiples of 4)? How do you know? Encourage the students to explain how they could build up or down by 3 (or 4).

Investigation

Which nine different numbers would you want to have on your multiplication facts Bingo card?

?	?	?
?	?	?
?	?	?

Think

- How does a game of Bingo work?
- If in a Bingo game, the caller called out multiplication facts (e.g. 3×4), would there be some numbers that would be better to have on your card?
- What range of numbers is possible for all of the multiplication facts?
- Are there numbers that could be called out more than once?
- Are there numbers between 1 (or 0) and 100 on the Bingo card that would never be called out?

Show and Do

Make an overhead transparency of this page. Reveal and read the Investigation question with the students. Encourage them to ask questions to clarify the task. If necessary, discuss the 'Think' prompts.

Ohserve

Was the student able to

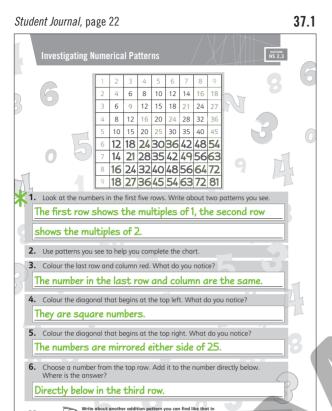
- recall or work out answers to basic multiplication facts?
- organise information to identify which totals (products) occur more than others in basic facts (composite numbers)?
- explain why certain numbers are better to include on their Bingo card and which ones should not be included?

Reflect

Compile a list of 'good' and 'bad' choices for numbers on the card. Use the data gathered to find the best numbers. (If you are including zero facts, including zero on the card will reinforce the zero rule in multiplication.) Have the class discuss where on the card to put the 'best' (i.e. most used) numbers.



ANSWERS



Revising the Use-a-Known-Fact Strategy

1. Colour the grid to match the first fact. Then write the answers.

a. b. $6 \times 6 = 36$ so $7 \times 6 = 42$ $5 \times 8 = 40$ so $6 \times 8 = 48$ c. $6 \times 10 = 60$ so $6 \times 9 = 54$ $3 \times 10 = 30$ so $3 \times 9 = 27$ 2. Write the answer to the first fact to help you answer the second fact.

b. $10 \times 7 = 70$ c. $10 \times 4 = 40$ so $9 \times 7 = 63$

Student Journal, page 24

* Answers will vary. This is one example.

Student Journal, page 25

37.3

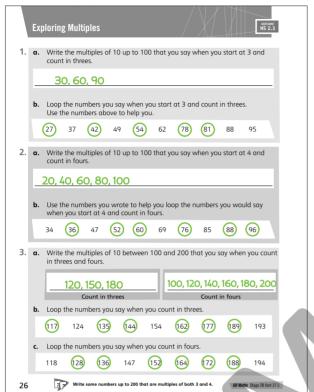
NS 2.3 Finding Factors Write all the factors of each number. 1 x 15 1 x 25 1 x 14 3_ x x 30 1 × 2_ x 1 x 36 1 x 80 1 x 100 2 x 50 x 25 × 8 x 10 x Write another pair of factors for 80 and 100. GO Maths Stage 2B Unit 37.4

37.4

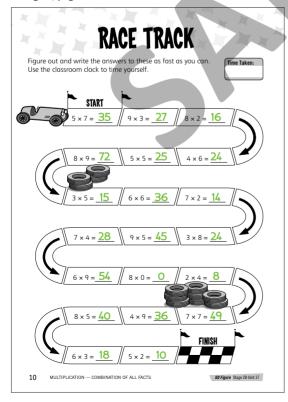
ANSWERS



Student Journal, page 26

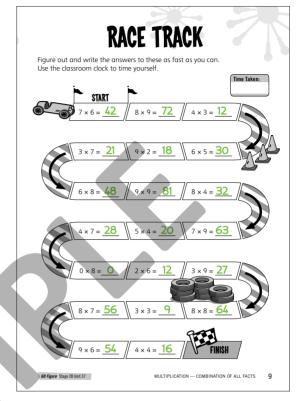


GO Figure, page 10

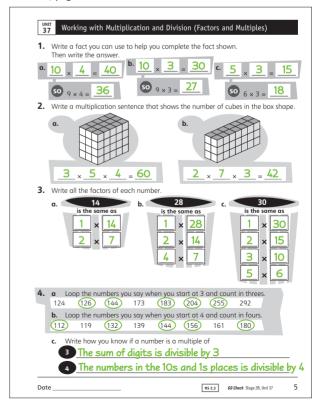


GO Figure, page 9

37.5



GO Check, page 5



UNIT 37 Materials and Resources

37.1

Investigating Patterns Involving Multiples

Materials

- GO Maths student journal, page 22
- Blank hundred chart copied onto an overhead transparency (use Blackline Master 4)
- Overhead projector
- Counters R

Optional

- 1 blank hundred chart (use Blackline Master 4) for each group of students
- 3 cubes R for each group of students with the following on the faces:
 - Cube 1: 1. 2. 3. 4. 5. 6
 - Cube 2: 3, 4, 5, 6, 7, 8
 - Cube 3: 4, 5, 6, 7, 8, 9

37.3

Multiplying with Three Factors

Materials

- GO Maths student journal, page 24
- Base-10 ones blocks

Optional

• 3 cubes from 37.1 for each group of students

37.2

Using a Strategy that Builds Up or Down from a Known Multiplication Fact

Materials

- GO Maths student journal, page 23
- 2 blank hundred charts (use Blackline Master 4)
- Box of Facts (Multiplication) Build-Up/Down Strategy Cards

Optional

- 1 blank hundred chart (use Blackline Master 4) for each group of students
- 3 cubes from the previous lesson for each group of students
- Counters R
- Fundamentals (Ages 9-10), pages 36-39

37.4

Investigating Methods to Find Factors

Materials

- GO Maths student journal, page 25
- GO Figure computation practice book, page 9
- Base-10 ones blocks or square tiles

37.5

Identifying Two-Digit and Three-Digit Multiples of 2, 3, 4 or 5

Materials

- GO Maths student journal, page 26
- GO Figure computation practice book, page 10
- Blank hundred chart copied onto an overhead transparency (use Blackline Master 4)
- Overhead projector
- Non-permanent marker

Investigation

Materials

• Overhead transparency of the Investigation question and 'Think' prompts